# Towards Robust Detection of Adversarial Examples

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## **Adversarial Examples**



Alps: 94.39%

Dog: 99.99%



Puffer: 97.99%



Crab: 100.00%

From Dong et al. (CVPR 2018)

**How to Defend Adversarial Attacks?** 

**Possible strategy one:** 

# To correctly classify adversarial examples

- Optimal
- Difficult to achieve
- Computationally expensive (adversarial training)

## **How to Defend Adversarial Attacks?**

**Possible strategy two:** 

# To detect and filter out adversarial examples

- Suboptimal
- Little computation
- Methods borrowed from anomaly detection

## We Detect Adversarial Examples, and How?

## **Design new detectors:**

- Kernel density detector (Feinman et al. 2017)
- LID detector (Ma et al. ICLR 2018)
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## We Detect Adversarial Examples, and How?

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Train the models to better collaborate with existing detectors

## **Reverse Cross Entropy**



$$\mathcal{L}_{CE} = -\mathbf{1}_{y} \cdot \log(\mathbf{F})$$

$$\mathcal{L}_{RCE} = -R_y \cdot \log(\mathbf{F})$$

**The RCE Training Method** 

**Phase 1: Reverse Training** Training the model by minimizing the RCE loss

**Phase 2: Reverse Logits** Negating the logits fed to the softmax layer to give predictions

## **Theoretical Analysis**

**Theorem 2.** (Proof in Appendix A) Let (x, y) be a given training data. Under the  $L_{\infty}$ -norm, if there is a training error  $\alpha \ll \frac{1}{L}$  that  $\|\mathbb{S}(Z_{pre}(x, \theta_R^*)) - R_y\|_{\infty} \leq \alpha$ , then we have bounds  $\|\mathbb{S}(-Z_{pre}(x, \theta_R^*)) - 1_y\|_{\infty} \leq \alpha (L-1)^2$ , and  $\forall j, k \neq y$ ,  $\|\mathbb{S}(-Z_{pre}(x, \theta_R^*))_j - \mathbb{S}(-Z_{pre}(x, \theta_R^*))_k\| \leq 2\alpha^2 (L-1)^2$ .

#### **Property 1: Consistent and Unbiased**

When the training error  $\alpha \rightarrow 0$ , the prediction tends to the one-hot label

#### **Property 2: Tighter Bound**

The difference between any two non-maximal elements decreases as  $O(\alpha^2)$ 

We first define the non-maximal entropy (non-ME) as:

nonME(x) = 
$$-\sum_{i\neq y} \widehat{F}(x)_i \log(\widehat{F}(x)_i)$$
,

where  $\widehat{F}(x)_i$  is the normalized non-maximal predictions.

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# RCE training encourages the maximal prediction to tend to 1, while maximizing the non-ME.



The left plot is the decision domain in 2d feature space for 3 classes (each class with one color)

When the non-ME of the returned predictions are maximized, the learned features for each class with tend to locate near the black dash lines, where the points on the dash lines have the maximal non-ME.



The left plot is the decision domain in 2d feature space for 3 classes (each class with one color)

When the non-ME of the returned predictions are maximized, the learned features for each class with tend to locate near the black dash lines, where the points on the dash lines have the maximal non-ME.



Then if an adversary want to craft an adversarial example based on  $z_0$ , he has to move further to  $z_2$  rather than  $z_1$  to obtain a normal value of non-ME.

Normal examples

Adversarial examples that succeed to fool detector

O Adversarial examples that fail to fool detector



In practice, the learned low-dimensional feature distributions by RCE make it more difficult to craft an adversarial examples with normal values of non-ME.

Table 1: Classification error rates (%) on test sets.

| Method          | MNIST | CIFAR-10 |
|-----------------|-------|----------|
| Resnet-32 (CE)  | 0.38  | 7.13     |
| Resnet-32 (RCE) | 0.29  | 7.02     |
| Resnet-56 (CE)  | 0.36  | 6.49     |
| Resnet-56 (RCE) | 0.32  | 6.60     |

#### **Classification error rates (%) on normal test examples**



#### t-SNE visualization of learned features on CIFAR-10



#### **Classification accuracy under different attack methods**

| Attool | Obj. | MNIST      |        | CIFAR-10        |            |        |                  |
|--------|------|------------|--------|-----------------|------------|--------|------------------|
| Анаск  |      | Confidence | non-ME | K-density       | Confidence | non-ME | K-density        |
| FGSM   | CE   | 79.7       | 66.8   | 98.8 (-)        | 71.5       | 66.9   | <b>99.7</b> (-)  |
|        | RCE  | 98.8       | 98.6   | <b>99.4</b> (*) | 92.6       | 91.4   | 98.0 (*)         |
| BIM    | CE   | 88.9       | 70.5   | 90.0 (-)        | 0.0        | 64.6   | <b>100.0</b> (-) |
|        | RCE  | 91.7       | 90.6   | <b>91.8</b> (*) | 0.7        | 70.2   | <b>100.0</b> (*) |
| ILCM   | CE   | 98.4       | 50.4   | 96.2 (-)        | 16.4       | 37.1   | 84.2 (-)         |
|        | RCE  | 100.0      | 97.0   | <b>98.6</b> (*) | 64.1       | 77.8   | <b>93.9</b> (*)  |
| JSMA   | CE   | 98.6       | 60.1   | 97.7 (-)        | 99.2       | 27.3   | 85.8 (-)         |
|        | RCE  | 100.0      | 99.4   | <b>99.0</b> (*) | 99.5       | 91.9   | <b>95.4</b> (*)  |
| C&W    | CE   | 98.6       | 64.1   | 99.4 (-)        | 99.5       | 50.2   | 95.3 (-)         |
|        | RCE  | 100.0      | 99.5   | <b>99.8</b> (*) | 99.6       | 94.7   | <b>98.2</b> (*)  |
| C&W-hc | CE   | 0.0        | 40.0   | 91.1 (-)        | 0.0        | 28.8   | 75.4 (-)         |
|        | RCE  | 0.1        | 93.4   | <b>99.6</b> (*) | 0.2        | 53.6   | <b>91.8</b> (*)  |

## AUC-scores ( $10^{-2}$ ) when detecting adversarial examples



Minimal distortion when applying the C&W attack under oblivious threat model, i.e., the adversary knows the classifier but does not know the detector

| Ohi  | MNIST |            | CIFAR-10 |            |
|------|-------|------------|----------|------------|
| 00j. | Ratio | Distortion | Ratio    | Distortion |
| CE   | 1     | 17.12      | 0        | 1.26       |
| RCE  | 77    | 31.59      | 12       | 3.89       |

Table 3: The ratios (%) of  $f_2(x^*) > 0$  and minimal distortions of the adversarial examples crafted by C&W-wb. Model is Resnet-32.

The results when apply the C&W attack under white-box threat model, i.e., the adversary also know the detector. The 'Ratio': the ratio of adversarial examples that induce higher values of detection metric than a threshold.



The visualization of the adversarial examples crafted by white-box C&W attack

|             | Res32 (CE) | Res32 (RCE) |
|-------------|------------|-------------|
| Res56 (CE)  | 75.0       | 90.8        |
| Res56 (RCE) | 89.1       | 84.9        |

Table 4: AUC-scores  $(10^{-2})$  on CIFAR-10. Resnet-32 is the substitute model and Resnet-56 is the target model.

AUC-scores ( $10^{-2}$ ) under the black-box threat model. We use the adversarial examples crafted on Resnet-32 to feed to Resnet-56