低精度推理

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《大模型计算》课程团队

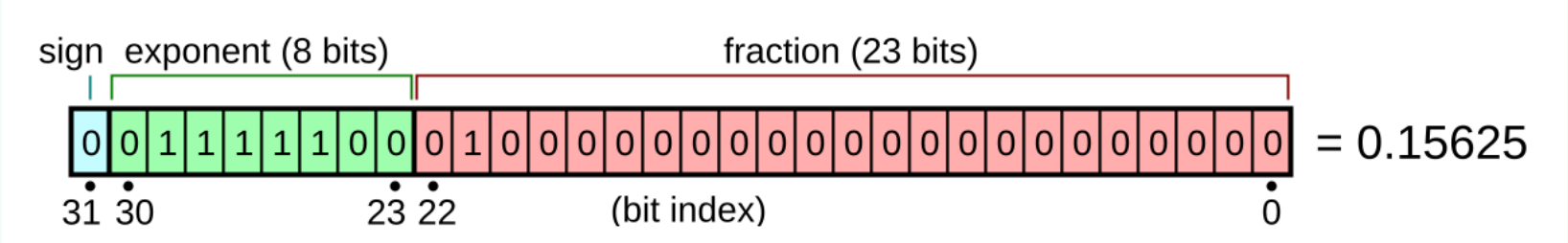
动机

	A100	H100	B200	RTX 4090	RTX 5090
BF16	312	1000	2250	165	209.5
FP8	N/A	2000 (2x)	4500 (2x)	330 (2x)	419 (2x)
INT8	624 (2x)	2000 (2x)	4500 (2x)	660 (4x)	838 (4x)
INT4	1248 (4x)	N/A	N/A	1320 (8x)	N/A
FP4	N/A	N/A	9000 (4x)	N/A	1676 (8x)

- BF16→FP4/INT4: 4～8倍加速
- FP16→INT1: 16倍存储空间节省

低精度数值格式

❖ IEEE 754



❖ **数值表示：**

$$(-1)^{s}\times(1.M)\times 2^{(E-B)}\text{(规格化数：E非全零)}$$

$$(-1)^{s}\times(0.M)\times 2^{(1-B)}\text{(非规格化数：E全零)}$$

❖ **对于FP32：** $B=127$

❖ **例中：** $(01111100)_2=124$，$(1.01)_2=1.25$，$1.25\times 2^{-3}=0.15625$

❖ **最小数值：** $2^{-23}\times 2^{-126}=2^{-149}\approx 1.4\times 10^{-45}$

❖ **最大数值：** $(2-2^{-23})\times 2^{127}\approx 3.4\times 10^{38}$

数值类型

数值	指数	尾数	最小值	最大值
FP32	8	22	1.2e-38	3.4e38
FP16	5	10	6e-8	65504
BF16	8	7	1.2e-38	3.4e38
FP8 (E5M2)	5	2	1.2e-38	5.7e4
FP8 (E4M3)	4	3	6.1e-5	448
FP4	2	1	0.5	6
INT8	0	7	-128 ~ 127	
INT4	0	3	-8 ~ 7	

低精度张量

❖ 高精度矩阵如何用低精度矩阵近似?

方法1: $\mathbf{X} \approx \mathrm{int}(\mathbf{X})$

❖ 问题：不同矩阵数值差异很大

❖ 矩阵A：[-1e-3，+1e-3]　　# 全零（下溢）

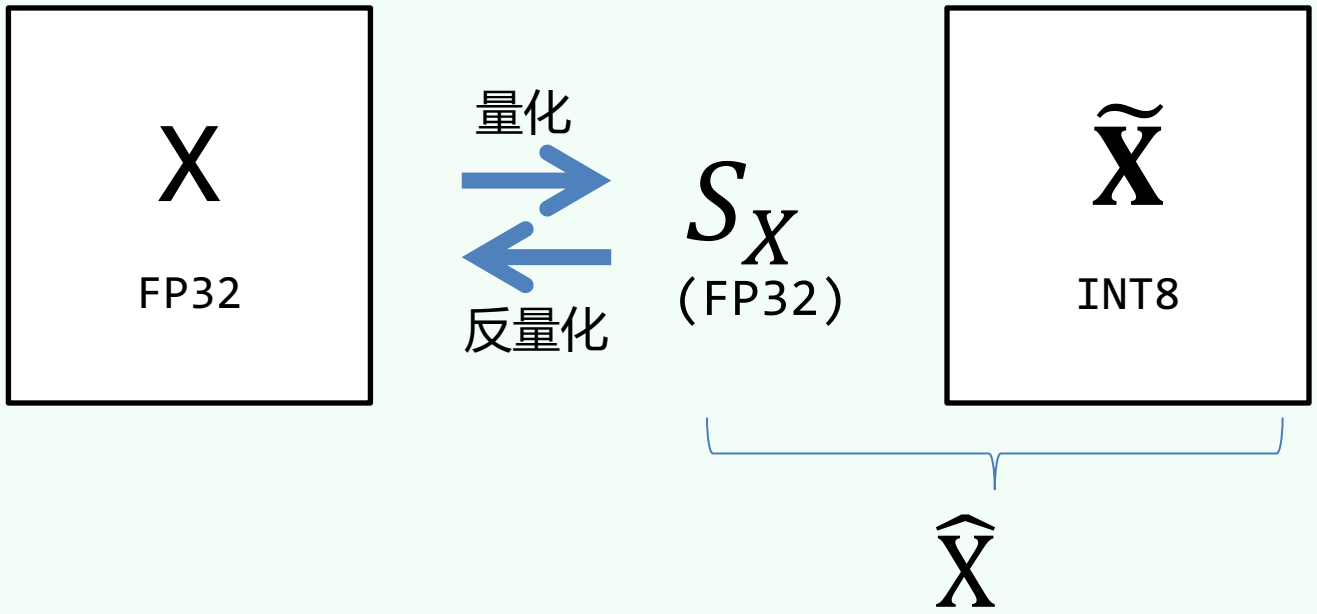
❖ 矩阵B：[-1e+4，+1e+4]　　# 全-128或+127（上溢）

❖ 解决方案：利用缩放因子（scaling factor）捕捉张量的数值范围

$$Q(\mathbf{X}) = s_x \times \mathrm{round}\left(\frac{\mathbf{X}}{s_x}\right) \approx \mathbf{X}$$

❖ $s_x = \max_{ij} |X_{ij}| / 127,\ \frac{X}{s_x} \in [-127, +127]$

量化与反量化




```
def quantize(X):
    s_x = max_ij |X_ij| / 127
    X̃ = round(X / s_x)
    return s_x, X̃
```

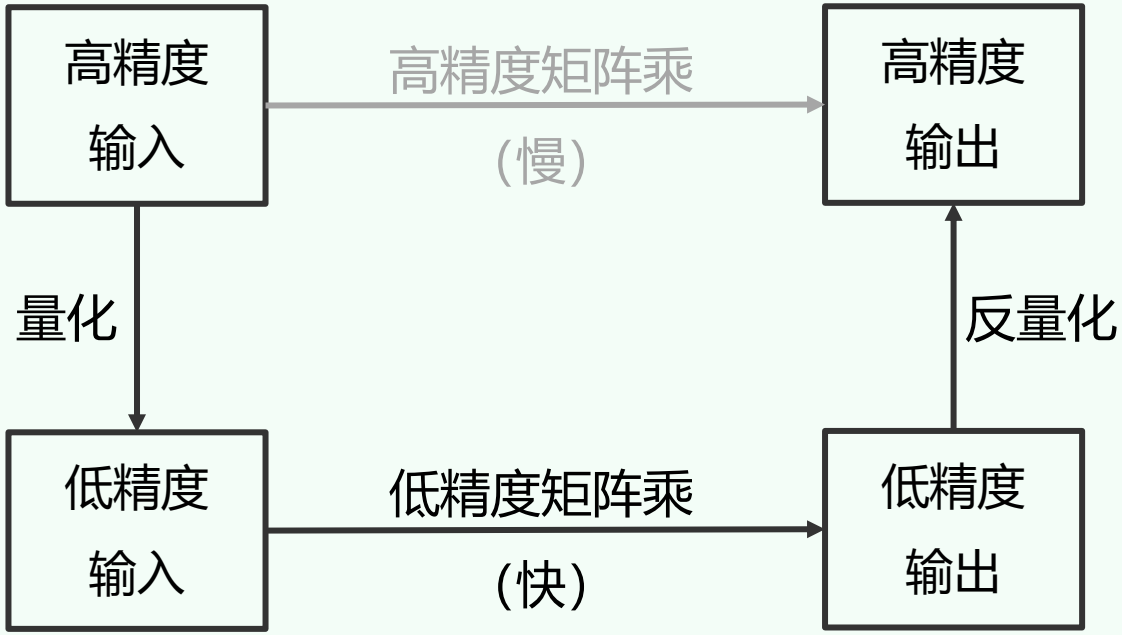
```
def dequantize(s_x, X̃):
    return s_x × X̃
```

低精度矩阵乘

$$\mathbf{X} \approx s_x \widetilde{\mathbf{X}}, \qquad \mathbf{Y} \approx s_y \widetilde{\mathbf{Y}}$$

$$\mathrm{XY} \approx (s_x s_y)\widetilde{\mathbf{X}}\widetilde{\mathbf{Y}}$$

```
def QGEMM(X, Y):
    s_x, X̃ = quantize(X)
    s_y, Ỹ = quantize(Y)
    Z̃ = X̃Ỹ        低精度矩阵乘（高效）
    return dequantize(s_x s_y, Z̃)
```



累加器

❖ $\tilde{Z}$的数值精度是什么?

$$\tilde{Z}_{ij} = \sum_{k=1}^{K} \tilde{X}_{ik}\tilde{Y}_{kj}$$

```
def QGEMM(X, Y):
    s_x, X̃ = quantize(X)
    s_y, Ỹ = quantize(Y)
    Z̃ = X̃Ỹ        低精度矩阵乘（高效）
    return dequantize(s_x s_y, Z̃)
```

```
for each i do
    for each j do
        z[i, j] = 0 ——→ 累加器 (accumulator)
        for each k do
            z[i, j] += x[i, k] * y[k, j]
                       └──────┘
                        INT16
```

❖ 对于INT输入，累加器均为INT32

❖ 对于FP输入，累加器均为FP32

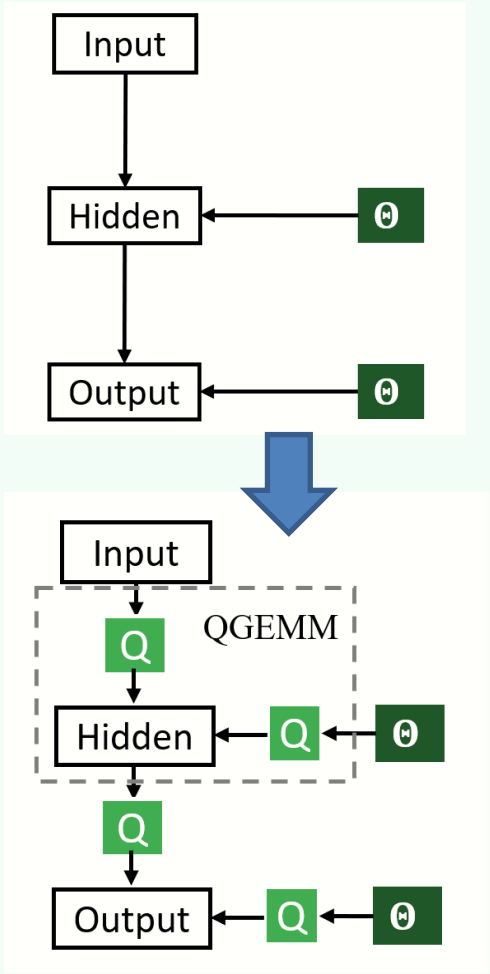
大模型中的量化

- 高度取决于要实现什么功能？对于每个场景，能达到的精度完全不同……

- 加速（串行）解码：权重量化、KV量化
- 加速（并行）解码/预填充：权重+激活量化，QKV量化
- 加速训练：权重+激活+梯度量化
- 降低KV缓存显存占用：KV量化

- 降低权重显存占用/模型存储空间：权重/优化器状态量化
- 降低KV缓存显存占用：KV量化
- 降低通信量：梯度量化

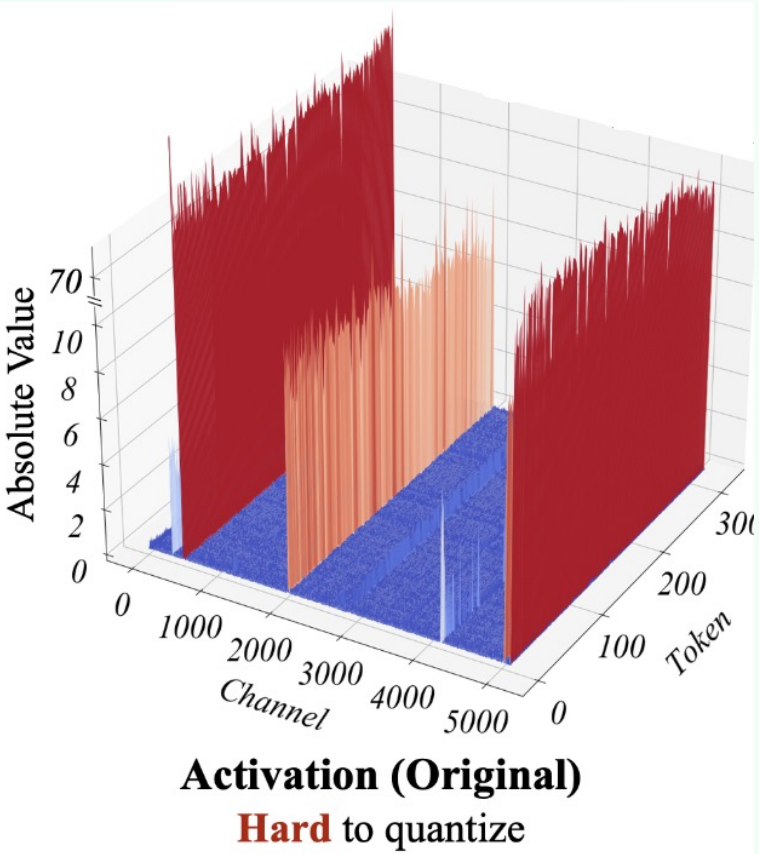
训练后量化（post-training quantization）



- 直接在训练完毕的模型中将 权重/激活 替换为量化后的版本
- 优势：方便快捷
- 劣势：准确率损失严重（俗称“掉点”）
  - 第一层输入/权重不准 造成 第一层输出不准
  - 因此第二层输入更加不准
  - 误差不断累积……
- 如何尽量不损失精度？两个线索
  - 让$Q(\mathrm{X}) - \mathrm{X}$尽量小
  - 让$Q(\mathrm{X})Q(\mathrm{Y}) - \mathrm{XY}$尽量小

离群值

- 实际的张量内数值差异巨大
- 最大值有时可达平均绝对值的10000倍



channel

token

0.01	1.02	0.16	100	10
0.03	0.31	2.42	300	5
0.32	1.23	1.63	200	6
0.16	1.53	0.32	400	15
0.24	2.13	1.32	1000	8

Round to zero
Huge information loss

0	0	0	104	0
0	0	0	304	8
0	0	0	200	8
0	0	0	400	16
0	0	0	1000	8

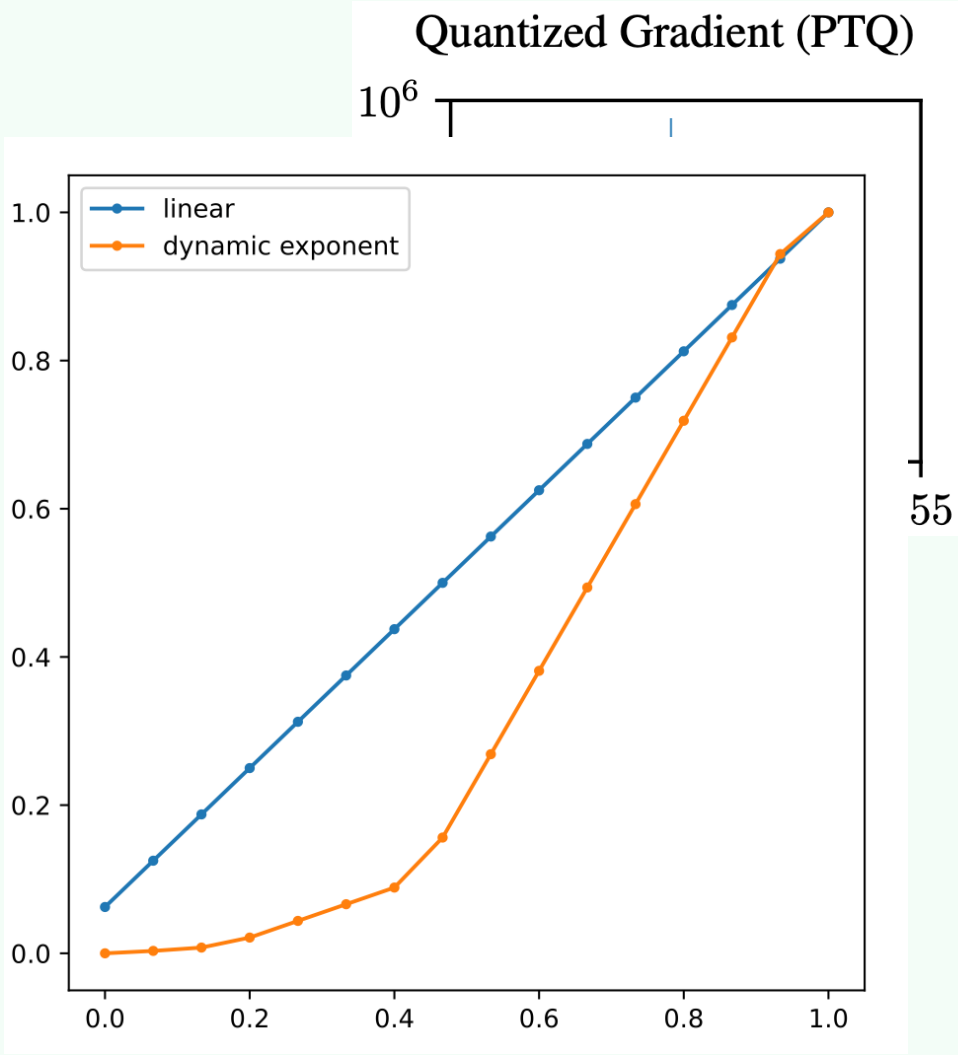
INT8 quantization: scales the largest element (1000) to +127

Resolution is $\frac{1000}{127} \approx 8$

解决方案1：非均匀量化

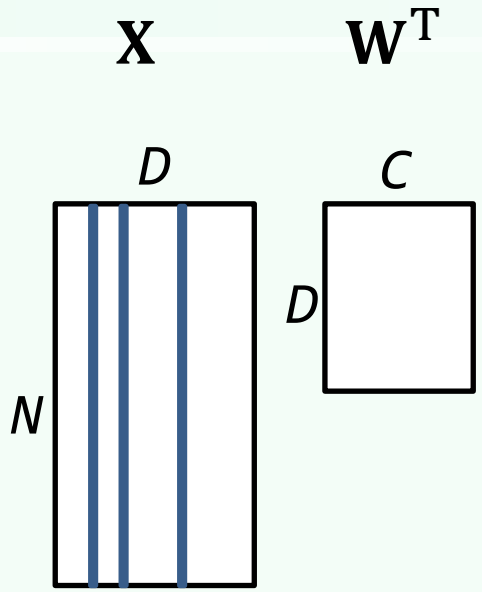
- 小数更多，大数更少
- 思路：接近0处量化点更多，远离0处量化点更少
- 整数：均匀
- 浮点数：非均匀

- 没有绝对的优劣，要看**数值格式**与**数值分布**是否一致
- 若数值分布非均匀：浮点更好
- 若数值分布均匀：指数更好

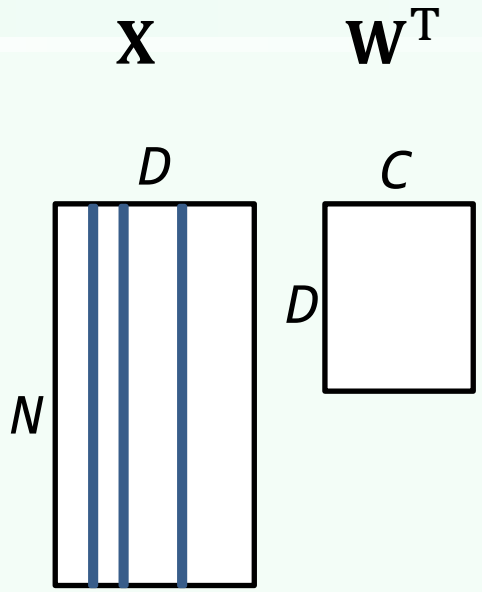


解决方案2：细粒度缩放

- 离群值模式
- $\mathbf{Y} = \mathbf{X}\mathbf{W}^{\mathrm{T}}$
- X：N词元 * D维度
- W：C通道 * D维度



解决方案2：细粒度缩放



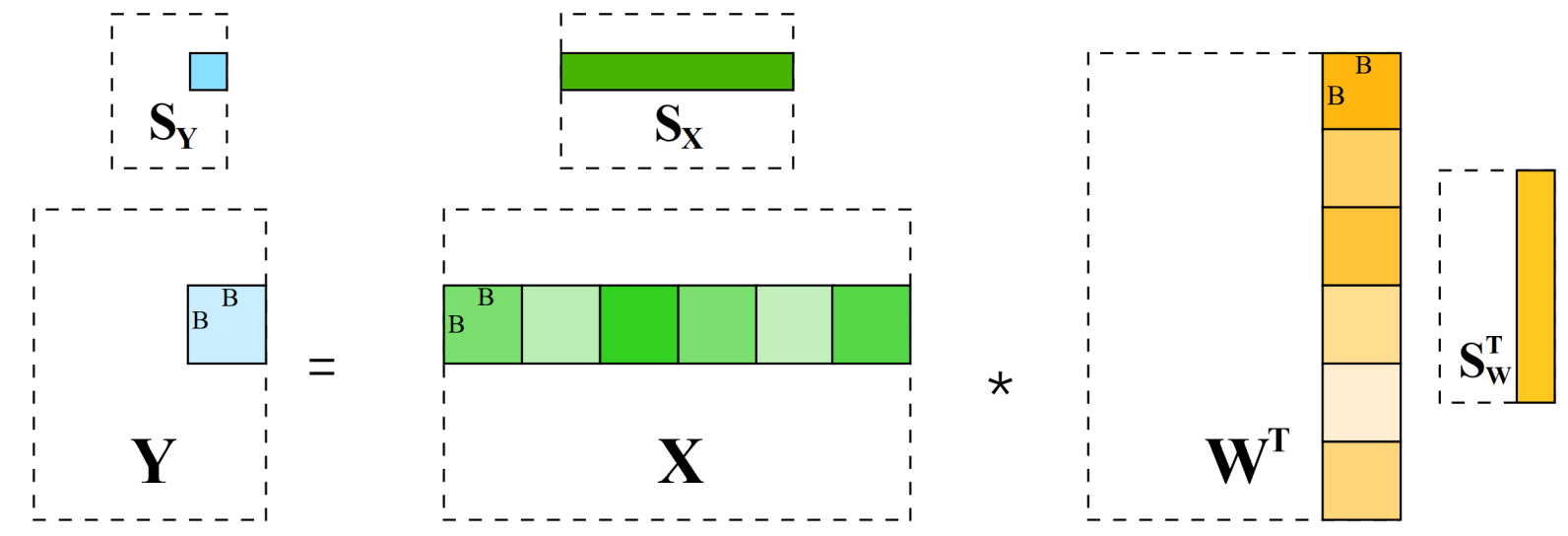
- **逐词元缩放：** $Q(\mathbf{X}) = \mathrm{diag}(\boldsymbol{s_x})\widetilde{\mathbf{X}}$
  - **缩放因子** $\boldsymbol{s_x} \in \mathbb{R}^N$
- **逐通道缩放：** $Q(\mathbf{W}) = \mathrm{diag}(\boldsymbol{s_w})\widetilde{\mathbf{W}}$
  - **缩放因子** $\boldsymbol{s_w} \in \mathbb{R}^C$
- **矩阵乘法** $Q(\mathbf{X})Q(\boldsymbol{W}) = \mathrm{diag}(\boldsymbol{s_x})\left(\widetilde{\mathbf{X}}\widetilde{\mathbf{W}}^T\right)\mathrm{diag}(\boldsymbol{s_w})$
- **思考：逐词元缩放并不能处理X的列状离群值，为什么不用逐维度缩放来完全排除离群值影响?**
- $Q(\mathbf{X}) = \widetilde{\mathbf{X}}\mathrm{diag}(\boldsymbol{s_x}), Q(\mathbf{X})Q(\boldsymbol{W}) = \left(\widetilde{\mathbf{X}}\mathrm{diag}(\boldsymbol{s_x})\widetilde{\mathbf{W}}^T\right)\mathrm{diag}(\boldsymbol{s_w})$
- $\left(\widetilde{\mathbf{X}}\mathrm{diag}(\boldsymbol{s_x})\widetilde{\mathbf{W}}^T\right)_{ij} = \sum_{k=1}^{D} X_{ik}W_{jk}(s_x)_k$
- **取决于目的：如果只是为了压缩X，可以；如果是为了加速XW^T，不行**

解决方案2：细粒度缩放—分块缩放

- X分成大小为(n，d)的块，每块为X_{ik}，缩放因子为s_{ik}
- W^T分成大小为(d，c)的块，每块为W_{kj}^T，缩放因子为t_{kj}
- 则$Y_{ij} = \sum_{k=1}^{K} (s_{ik} t_{kj}) (X_{ik} W_{kj}^T)$
- 计算流程：全局累加器 += 反量化（张量内核累加器）

Jetfire: n=d=c=32

DeepSeekv3: n=1, d=c=128



解决方案2：细粒度缩放—微缩放（microscaling）

MXFP数值格式：n=c=1，d=32

缩放因子为UE8M0

- MXFP8
- MXFP6（速度同FP8）
- MXFP4（速度同FP8）

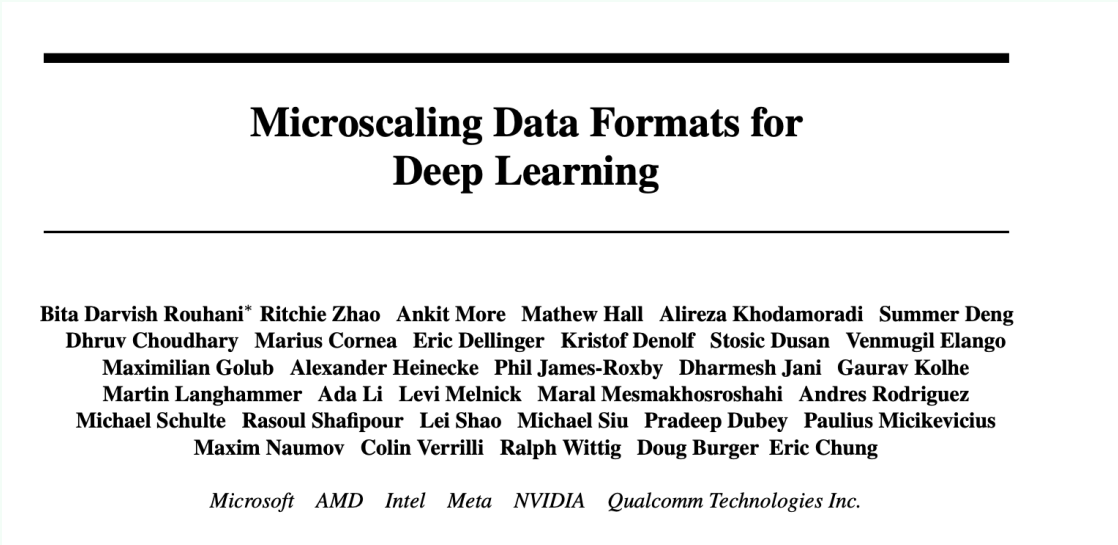
NVFP数值格式：n=c=1，d=16

缩放因子为E4M3

- NVFP6（速度同FP8）
- NVFP4（2～4x快于FP8）

Nvidia Blackwell起支持

几乎完全解决了离群值的问题，且自带反量化，计算很快



Microscaling Data Formats for Deep Learning

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Maxim Naumov Colin Verrilli Ralph Wittig Doug Burger Eric Chung

Microsoft AMD Intel Meta NVIDIA Qualcomm Technologies Inc.

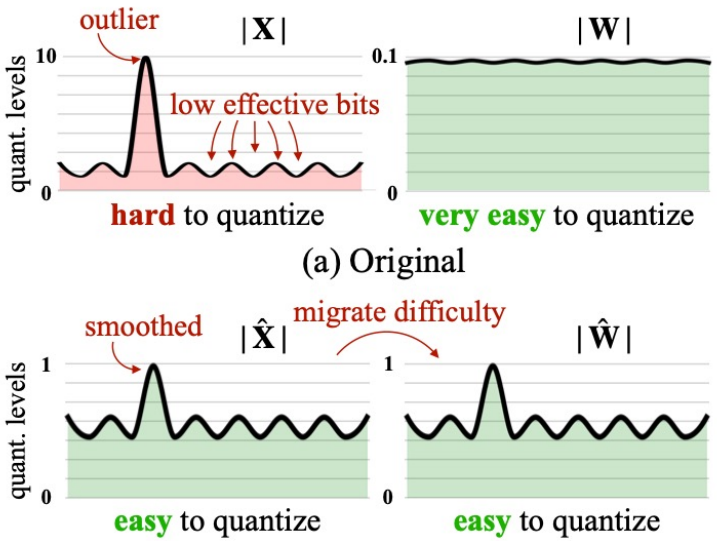
解决方案3：旋转—SmoothQuant

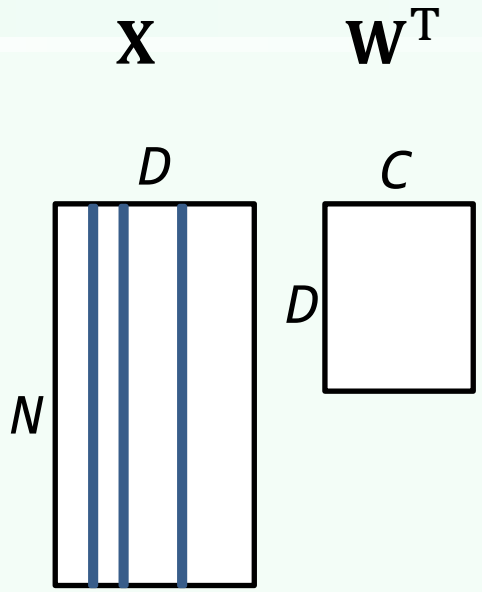
- 主要发现：

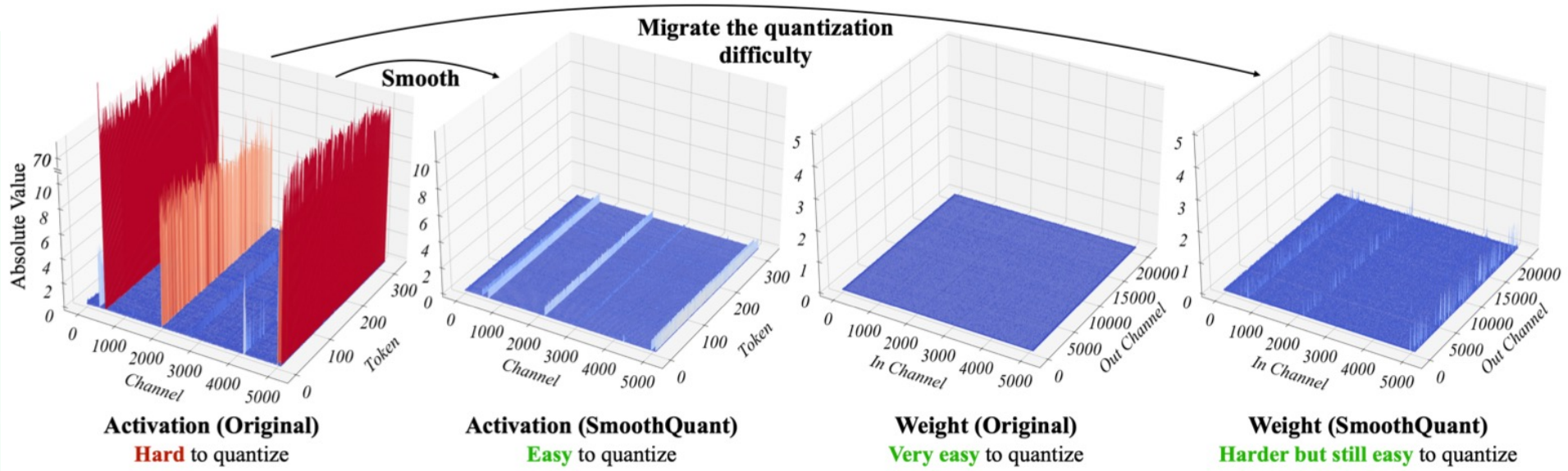
$$\mathrm{X}\mathrm{W}^{T} = \mathrm{X}PP^{T}\mathrm{W}^{T} \approx Q(XP)Q(P^{T}\mathrm{W}^{T})$$

- 可以在其他子空间做量化
- 量化难度可以在X和W间分配

解决方案3：旋转—SmoothQuant







解决方案3：旋转—哈达马变换

- 考虑特例：X矩阵（NxD）只有一列很大（0～10000），其他列均很小（0～1）
- 量化误差正比于缩放因子
- 直接量化：误差为O(ND)
- 逐列量化：误差为O(N)—离群值列不会“牵连”其他列
- 但是绝大部分的比特都用来存储非离群值，而这些数都接近零！
- 宝贵的比特浪费在存储无信息量的零上面！ # 离群值究竟是否更有信息量？为什么？
- 哈达马变换：将离群列的信息均摊到其他列上
- 误差为O(N/D)

0.01	1.02	0.16	100	10
0.03	0.31	2.42	300	5
0.32	1.23	1.63	200	6
0.16	1.53	0.32	400	15
0.24	2.13	1.32	1000	8

- 所谓高效深度学习，是要理解神经网络如何工作，并简化其计算，在不失功能的情况下提升效率

哈达马(Hadamard)矩阵

$$H_1 = [1],$$

$$H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix},$$

$$H_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix},$$

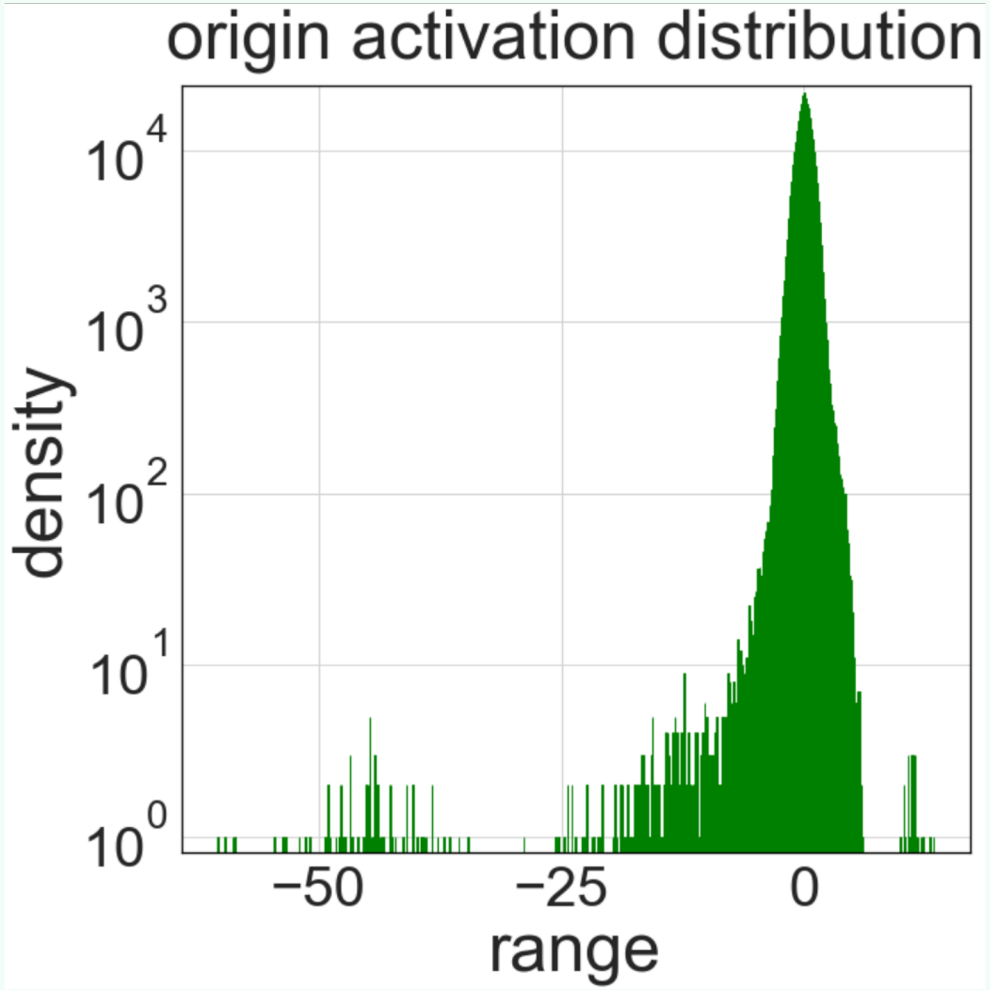
$$H_{2^k} = \begin{bmatrix} H_{2^{k-1}} & H_{2^{k-1}} \\ H_{2^{k-1}} & -H_{2^{k-1}} \end{bmatrix}$$

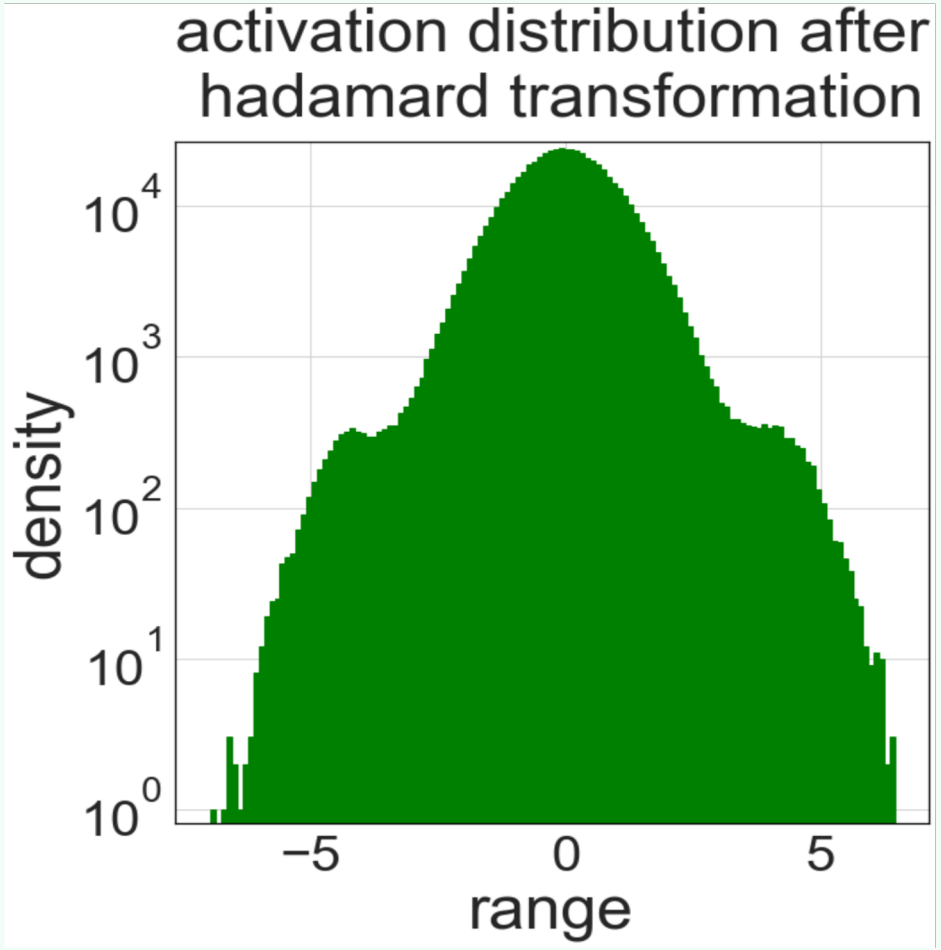
- 性质1：哈达马矩阵为对称正交阵：$HH = \mathbf{I}$
- 性质2：哈达马矩阵将单位向量转为全一向量，反之亦然

$$H_N e_i = (\pm 1, \pm 1, \ldots, \pm 1)/\sqrt{N}$$

- 因此$\boldsymbol{X}H_N$将是一个理想的，无离群值的矩阵
- 将代表示的离群值均为分布到了各个维度，每维贡献一点
- 量化$Q(\mathbf{X}) = \text{round}(\boldsymbol{X}H)H$
- 矩阵乘法$\mathbf{XW} = \mathbf{XHHW} \approx Q(\boldsymbol{XH})Q(\boldsymbol{HW})$
- 恰好处理了逐列的离群值
- 哈达马变换仅用于预处理，几乎无实现开销

解决方案3：旋转—哈达马变换

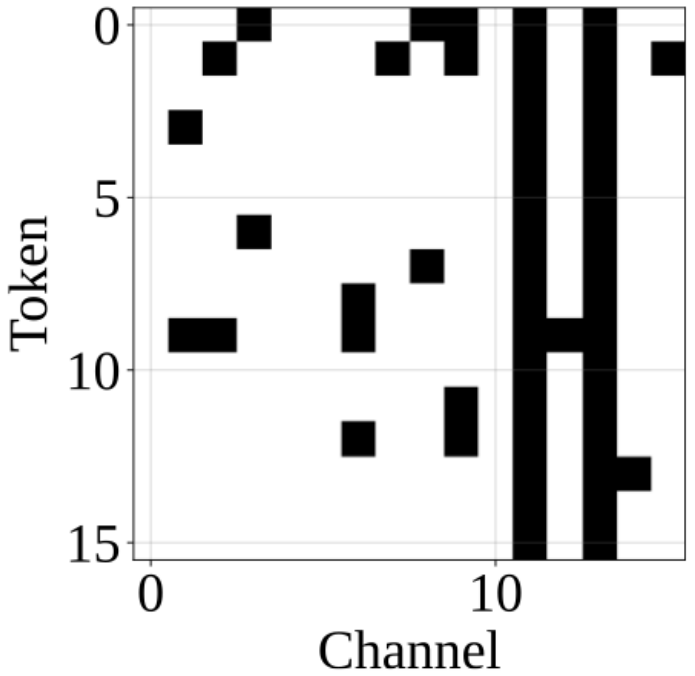




解决方案4：混合精度

- 这一列这么重要，为什么不干脆设成更高精度?
- 误差为$O(N2^{-D})$

0.01	1.02	0.16	100	10
0.03	0.31	2.42	300	5
0.32	1.23	1.63	200	6
0.16	1.53	0.32	400	15
0.24	2.13	1.32	1000	8



目前是什么状态?

- 采用了NVFP4量化以后，对神经网络中的张量进行量化，损失和对随机张量量化差不多

- 可能已逼近信息论极限，各种技术均不会再有太大效果

大模型量化实践

清华大学计算机系 陈键飞

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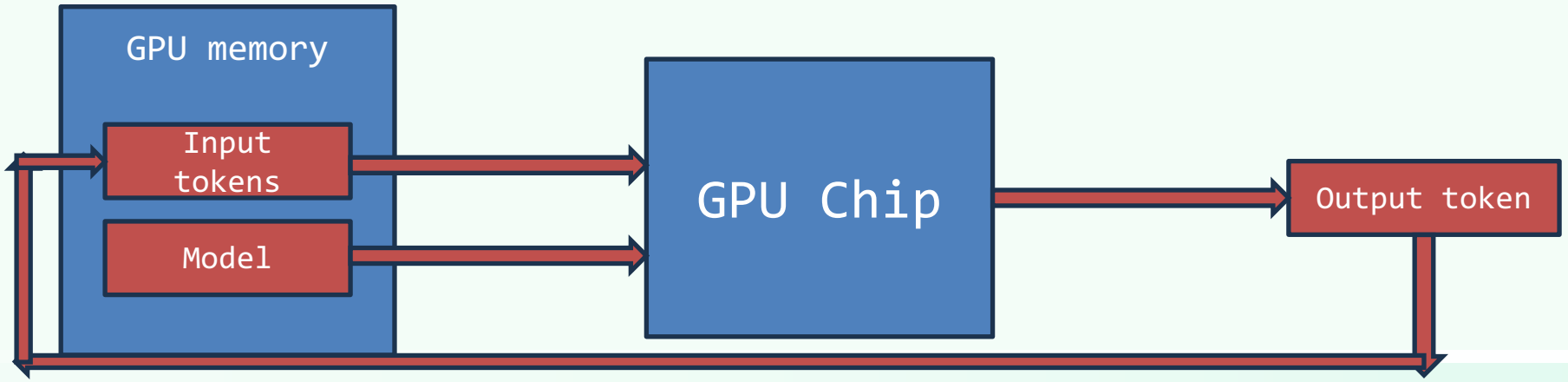
仅权重量化

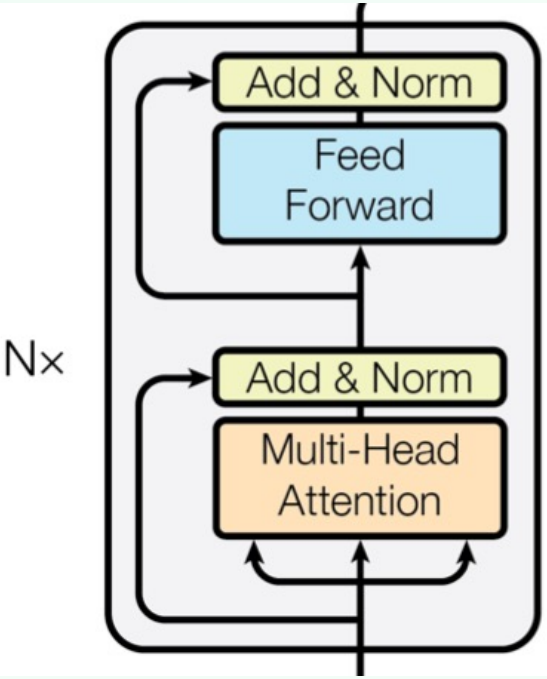
- $\mathbf{X}\mathbf{W}^{\mathrm{T}} \approx X Q(\mathbf{W}^{\mathrm{T}})$
- 之前非均匀量化/旋转/混合精度等各种trick被用来提点，现在直接转NVFP4通常损失已可接受
- 模型越大，掉点越少（为什么？）
- 访存共计15GB / token（简易算法：每个参数2字节）
- 理想情况下，如果打满带宽，解码速度为1008 / 15 = 67 token/s
- 采用4比特量化，访存除4，速度乘4



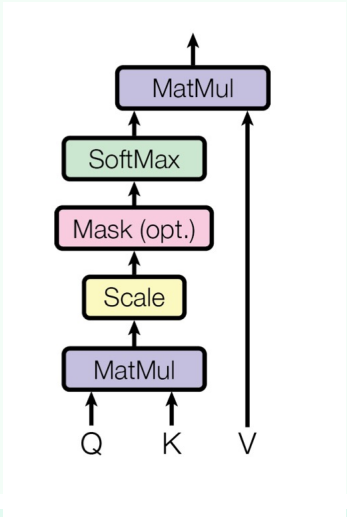
权重+激活量化

- SmoothQuant可以实现pertensor的WINT8 AINT8量化
- 直接转NVFP4损失稍大
- SVDQuant可能可以实现NVFP4量化，有待尝试

Low-precision Attention

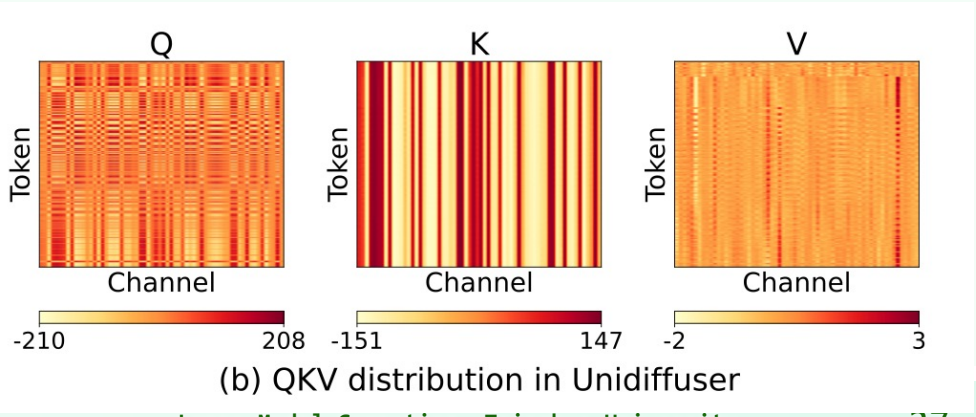


- Attention mostly remain in FP/BF16
- Long context tasks: text summarization / video generation
- MM1: $P = softmax(QK^T /sqrt(d))$
- MM2: O = PV
- Goal: quantize Q, K, P, V



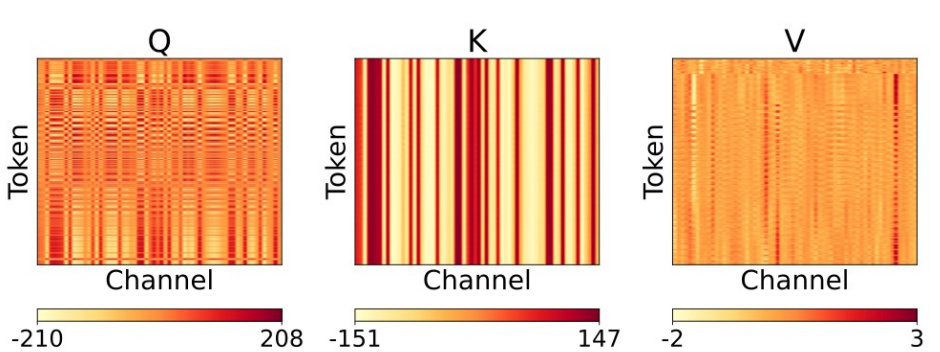
Challenges

- Problem 1: QK has large bias
- Problem 2: the logits qK^T = large constant + (small per-dim variations)
- Problem 3: the probability P has many small quantities, but the overall contribution is large
- Problem 4: Ada and Hopper's FP8 tensor core only has 22-bit accumulator
- C(FP22) = C(FP22) + A(FP8)B(FP8)



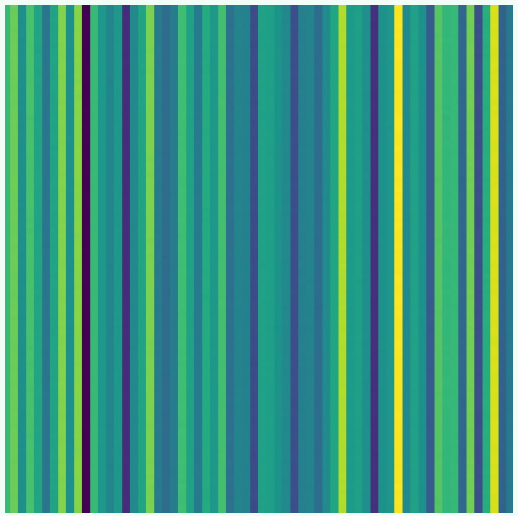
(b) QKV distribution in Unidiffuser

SageAttention

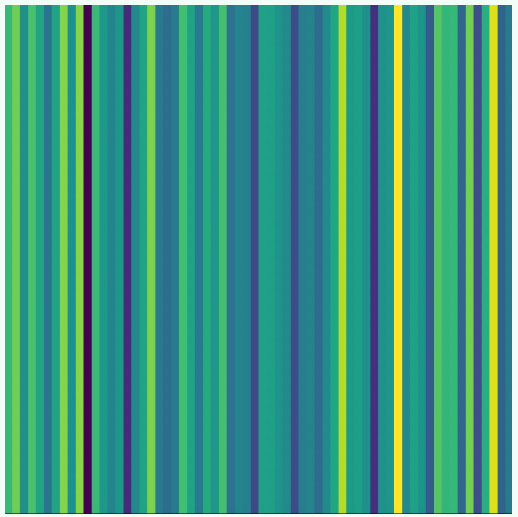


(b) QKV distribution in Unidiffuser

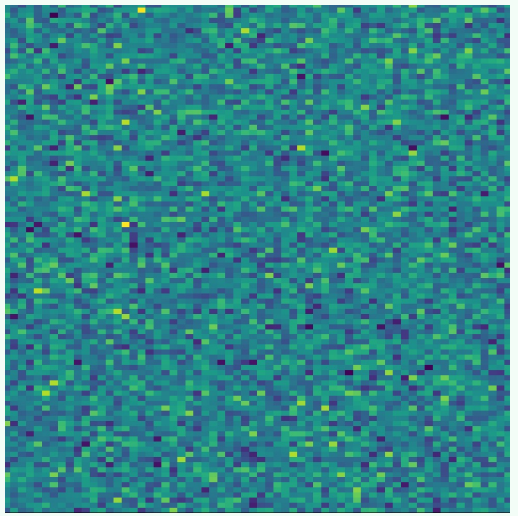
$$K = k + \Delta K$$



$=$



$+ 0.01*$



$$\text{softmax}(qK^T) = \text{softmax}(qk^T + q\Delta K^T) = \text{softmax}(q\Delta K^T)$$

SageAttention

Table 2: **Average accuracy** using different data types across all layers of real models.

Q, K	$\widetilde{P}, V$	Cos Sim ↑	Relative L1 ↓	RMSE ↓
INT8	E4M3	99.94%	0.0345	3.53e-3
	E5M2	99.81%	0.0572	6.11e-3
	INT8	99.70%	0.1035	6.82e-3
E4M3	E4M3	99.81%	0.0607	5.93e-3
	E5M2	99.68%	0.0769	7.72e-3
	INT8	99.58%	0.1199	8.31e-3
E5M2	E4M3	99.37%	0.1107	1.09e-2
	E5M2	99.22%	0.1213	1.20e-2
	INT8	99.13%	0.1583	1.24e-2

❖ **SageAttention1**

➢ Surpress K outlier by substracting mean

$$\gamma(K) = K - \text{mean}(K)$$

$$\sigma(q(K - \text{mean}(K)^\top)) = \sigma(q\hat{K}^\top - q \cdot \text{mean}(K)) = \sigma(q\acute{K}^\top)$$

❖ **SageAttention2**

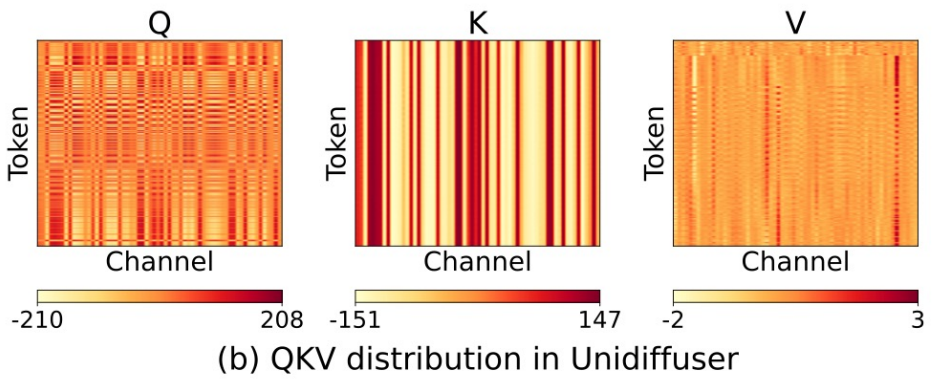
➢ Solution: further smooth K and V by subtracting the mean

❖ **SageAttention3**

➢ Utilize NVFP4 for QKPV with two-level scaling

❖ **Fine-grained dynamic quantization**

❖ **Use INT rather than FP for QK**



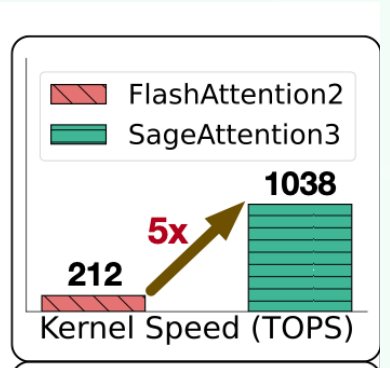
(b) QKV distribution in Unidiffuser

Zhang, Jintao, et al. "SageAttention: Accurate 8-Bit Attention for Plug-and-play Inference Acceleration." *ICLR 2025.*

Zhang, Jintao, et al. "SageAttention2 Technical Report: Accurate 4 Bit Attention for Plug-and-play Inference Acceleration." *ICML 2025.*

Results

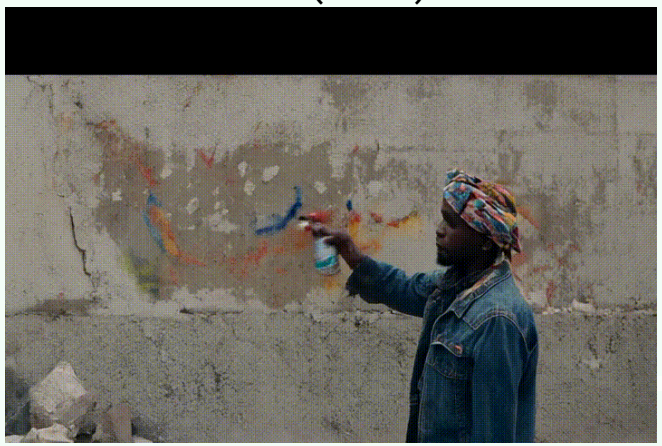
- FlashAttention2: FP16 attention (~208 Tflops on RTX 4090)
- SageAttention1 recipe: (~473 Tflops)
  - INT8 QK (4x speedup)
  - FP16 PV with FP16 accumulator (2x speedup)
- SageAttention2 recipe: (~537 Tflops)
  - INT4 QK (8x speedup)
  - FP8 PV with FP22 accumulator (2x speedup)
- SageAttention3 recipe: (~1022 Tflops)



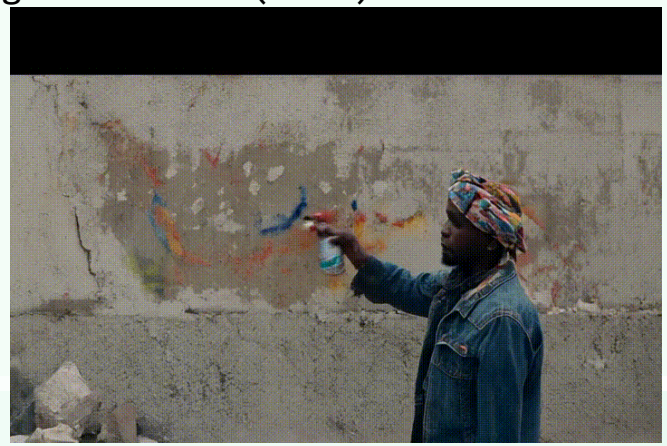
HunyuanVideo

540p, 4s

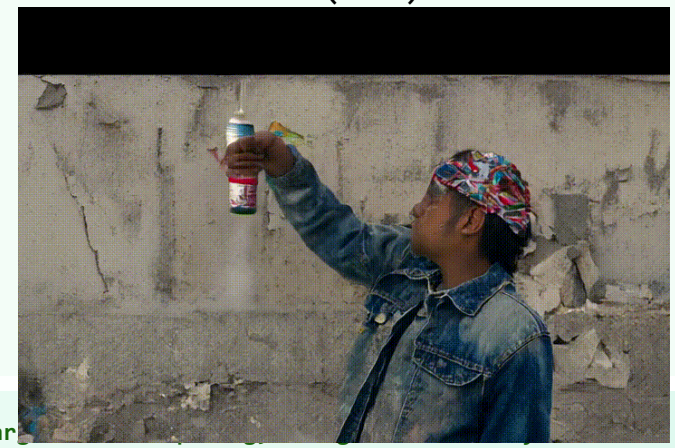
FlashAttention 2 (FP16)



SageAttention (INT8)



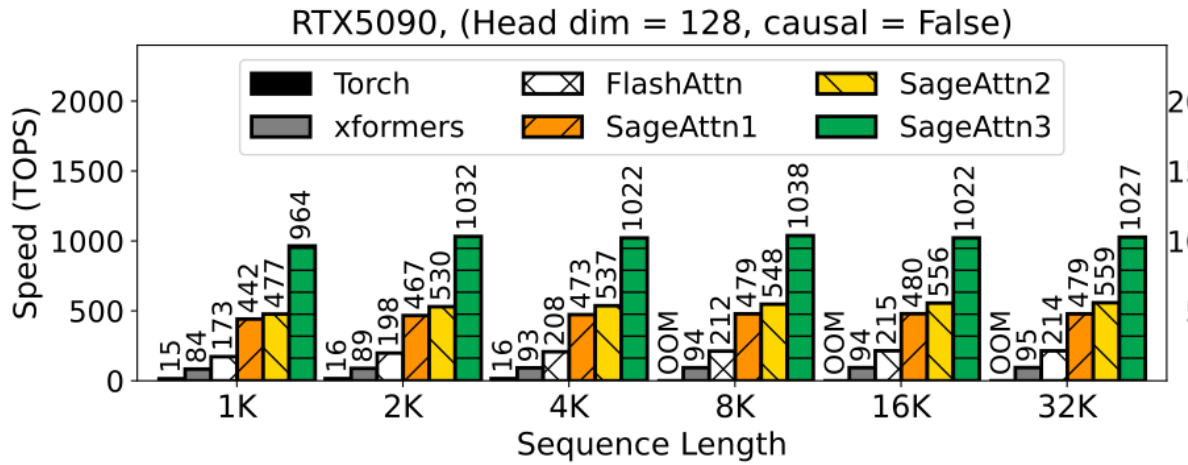
FlashAttention 3 (FP8)

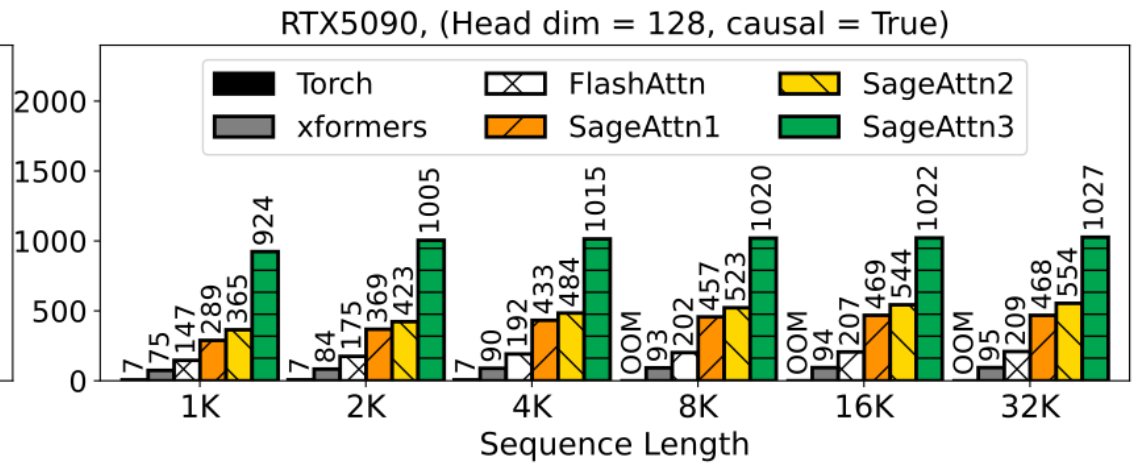


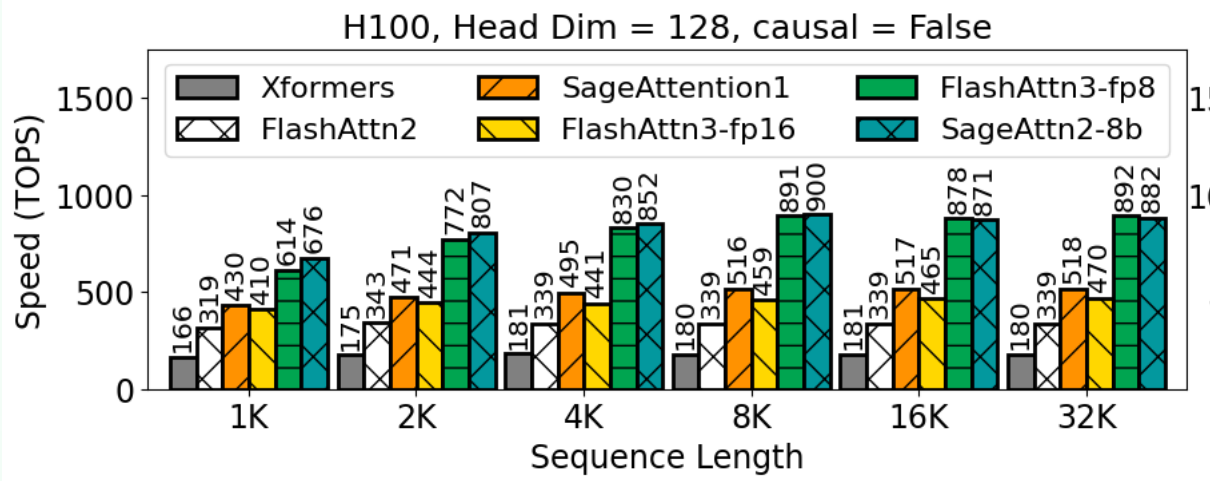
Results

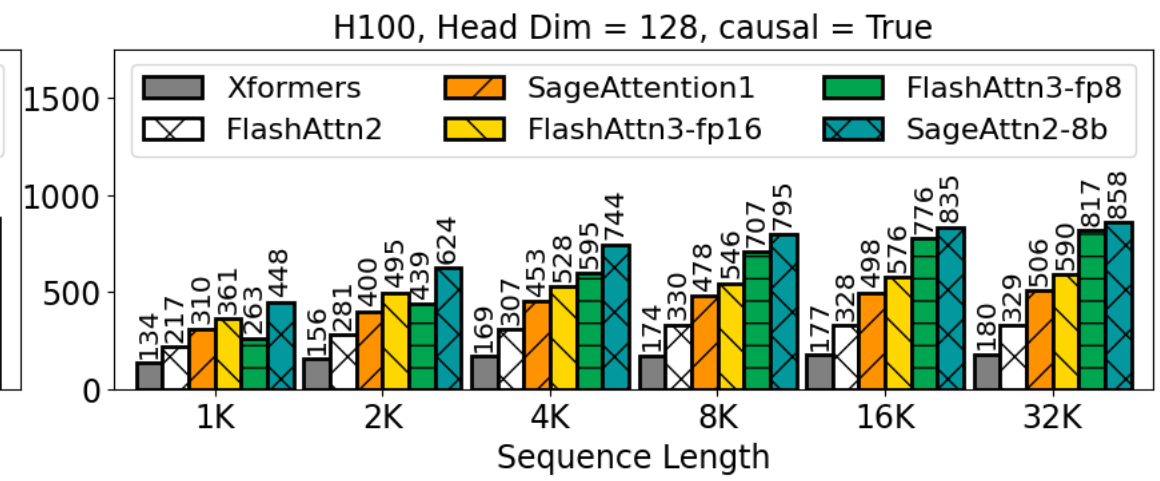
- SageAttention3, 1K seqlen, 964 Tflops @ RTX5090
- FlashAttention3, 32K seqlen, 892 Tflops @ H100

5090 beats H100!!! 10x cheaper!!!









KV量化

- 注意到只有KV需要量化，Q不需要量化
- Again，旋转。把困难的问题都留给Q
- $S = QK^T = \left(Q\mathrm{diag}(s)\right)(\mathrm{diag}(s)^{-1}K^T)$

- 目前可以做到4-bit KV
- Llama-3-8B, seqlen=8192
- KV缓存：1GB
- 4比特量化后：256MB
- 16GB显存即可做到batchsize=64，配合权重量化理论上足够打满GPU算力

谢谢

清华大学计算机系 陈键飞

《大模型计算》课程团队